Formation mechanisms of radial electron fluxes in a positive column

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The main physical formation mechanisms of differential radial electron fluxes (i.e., of the fluxes which correspond to different parts of the distribution function) in a dc positive column are discussed. It is shown that the magnitude and even sign of these fluxes depend crucially on whether the electron distribution is local or nonlocal and on electron energy. In the case of a nonlocal electron distribution the flux of slow electrons in the body of the distribution function is directed outward (to the wall), and for the fast electrons both directions are possible. The flux directed outward results from escape of the electrons to the tube wall, and the flux directed inward is caused by inelastic collision with large energy losses. In the local case the flux of the fast electrons, with the exception of a small wall vicinity, is directed inward, and the fluxes of the electrons in the distribution body are directed both inward and outward depending on the energy relaxation law. [S1063-651X(99)01208-8]

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I. INTRODUCTION

Since basic characteristics of gas discharge plasmas, especially at moderate and low pressures and moderate energy input, cannot be understood properly in the traditional framework of a fluid approach, in the last decades interest in detailed kinetic descriptions increased strongly. In realistic situations both the sources of particles and the energy input are spatially inhomogeneous. The straightforward solution of such a complicated problem in a real geometry can be performed only numerically. For an understanding of the underlying physics, for a solution of complex self-consistent problems, and for everyday laboratory practice simplified approaches are very desirable.

The so-called ''local approximation'' is valid for cases of smooth inhomogeneities. It is based on the assumption that in zeroth order the situation is well described by the electron distribution function (EDF) in spatially homogeneous plasmas, which is determined by local values of such macroscopic parameters as electric field strength, plasma density, etc. The inhomogeneity results in small corrections which determine particle and (if necessary) energy fluxes. For strongly inhomogeneous cases, when the electron spatial displacement occurs faster than its evolution in energy, a nonlocal approach which was proposed in [1,2] is very effective. The EDF in this case depends practically only on the total energy,

$$\varepsilon = \frac{mv^2}{2} + e\Phi(\vec{r}), \tag{1}$$

where $\Phi(\vec{r})$ corresponds to the space charge potential and e is the negative electron charge. The validity and accuracy of these approaches were investigated in detail in [3].

Spatial electron fluxes, which arise in inhomogeneous plasmas in nonuniform external fields, represent interesting and important plasma characteristics. They result in a redistribution of particles and energy over the plasma volume, in the formation of plasma density profiles, and they contain information of such subtle discharge features as plasma loss and generation, energy balance mechanisms, formation mechanisms of the EDF, etc. It is clear that in the local limit, when the EDF is formed predominantly due to processes of momentum and energy balance in a given place, the EDF is determined by local parameters. The small corrections to the local EDF which are proportional to the relatively small gradients of these parameters manifest themselves in the formation of fluxes of particles and energy which are proportional to these gradients. Such an approach is characteristic for the plasma transport theory (the so-called fluid approach; see, for example, [4,5]). In this approximation the direction of the differential fluxes (i.e., of the contributions of different EDF parts to the total particle and energy fluxes) coincides with the direction of the total fluxes. As the plasma becomes more inhomogeneous, the fluxes, generally speaking, increase, and the differential fluxes become more ''independent.'' The concept of differential fluxes turned out to be rather efficient in current-carrying plasmas [6]. In this paper it will be applied to the plasma of the positive column, in which the current flows only across the plasma inhomogeneity. In recent publications [7,8] the surprising results of numerical modeling were reported for the simplest case of a dc positive column in inert gases. The differential radial electron fluxes in different regions of phase space behaved themselves in a strikingly different manner. Even the directions of these fluxes at the same spatial point were often opposite at different energies. At low energies these fluxes were directed outward (positive). In contrast, at higher energies, which correspond to the EDF tail in [7], they turned out to be negative. Using more realistic boundary conditions at the tube wall in [8], it was found that at the plasma periphery (and at higher energies even in the whole plasma volume) the differential radial fluxes remain positive. Only in a restricted region of phase space are the fluxes in these calculations directed inward. These facts totally contradict the traditional fluid approach, which implies that in nonuniform plasmas the whole EDF drifts in the same direction [7,8]. Below an interpretation of these results is presented. It is demonstrated that the

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radial electron fluxes are formed as a result of the competition and superposition of several different physical mechanisms. The amplitude and even the sign of the resulting differential flux are determined by their relative magnitude. Accordingly, the integral fluxes of particles and energy cannot be expressed in terms of macroscopic parameters. Different possible scenarios when one of these mechanisms dominates and corresponding criteria are sketched.

In the case when at least the EDF body is nonlocal, i.e., depends on the total energy ε, the differential electron flux of the body electrons is directed outward due to expansion of the area which is available for an electron of a given ε. The outward-directed fluxes of fast electrons of the distribution tail are caused by their escape to the tube walls. They dominate at the column periphery at values of ε which exceed the wall potential. In the tube center, inward-directed fluxes exist. They result from the fact that the inelastic collisions of an electron of a given ε occur mainly in the central region due to a sort of ''diffusive heating'' in the ambipolar electric field.

II. GENERAL CONSIDERATIONS

At relatively low pressures, when at least for the majority of electrons the nonlocal approximation is valid, attempts to use any modifications of the fluid approximation [9–12] are misleading.

In the specific case of a dc positive column, the total radial electron flux, due to the ambipolarity condition, is extremely small. In the nonlocal case this flux consists of electrons with total energy ε higher than the wall potential Φ_w. In other words, the small total ambipolar electron flux is transported in the form of free diffusion of a small fraction of high-energy electrons with $\varepsilon > e\Phi_w$. The resulting EDF tail depletion is restored by the energy diffusion (i.e., by the Ohmic heating) of the bulk plasma electrons with $\varepsilon < e\Phi_w$.

The differential radial fluxes of these latter ones are caused by two dominant mechanisms. The first of them is connected to the expansion of the area which is available for an electron (of energy ε) with ε. Since the energy losses in elastic collisions at moderate and low pressures, when the nonlocality concept is valid, are usually small, the energy diffusion leads to a growth of ε, and this mechanism results in the outward-directed differential fluxes. As a result, the outward-directed flux of the electrons of the EDF ''body'' with $\varepsilon < \varepsilon_1$ (in atomic gases ε_1 corresponding to the excitation energy of the first level) arises.

The inelastic collisions with large energy losses $\geqslant \varepsilon_1$ occur mainly in the central region of the discharge tube and result in a considerable depletion of the EDF tail at $\varepsilon > \varepsilon_1$ and in differential fluxes directed inward at $\varepsilon_1 < \varepsilon < e\Phi_w$. These fluxes were interpreted in [13] as a manifestation of the effective diffusive heating. It is easy to see that the inward-directed differential fluxes, which are connected to the processes of excitation and ionization, are counterbalanced by the outward-directed ones at lower energies (which are caused by an expansion of the available area) and by the electron outflux to the tube wall at energies above the wall potential. The resulting energy flux can be directed both outward and inward. It is inward directed if the excitation rate considerably exceeds the ionization rate. On the other hand, since the wall potential in the nonlocal regime exceeds the ionization potential, if the excitations are rare, the energy flux is directed outward, the summary effect corresponding to the diffusive cooling. It follows that both phenomena of diffusive cooling [14] and of diffusive heating are of kinetic nature in essence.

At higher pressures the deviations from the nonlocality become significant. They are more pronounced at the EDF tail $\varepsilon > \varepsilon_1$. In a considerable pressure interval, the EDF ''body'' at $\varepsilon < \varepsilon_1$ remains nonlocal, but the tail decay is determined by the local field. In this case the flux at $\varepsilon < \varepsilon_1$ is directed outward and at $\varepsilon_1 < \varepsilon < e\Phi_w$ directed inward. The situation at $\varepsilon > e\Phi_w$ is more complicated. In the wall vicinity the differential fluxes are, of course, directed outward. But in [8] it was demonstrated that in the central tube region both directions are possible. A further increase of pressure results in a situation that the whole EDF becomes mainly local. In this case it is possible, as it is usually in the nonlocal case, to neglect the energy losses by elastic (and quasielastic [6]) collisions with relatively small energy loss so that the differential fluxes at kinetic energy below the average electron energy are outward directed, too. At higher energies they change sign and become inward directed. Only in the close vicinity of the wall (of the order of the electron energy relaxation length) is the resulting outward-directed flux of fast electrons, which is equal to the ion flux, formed. But at such high pressures, as a rule, the energy balance of the electron gas is determined by the elastic (or quasielastic [6]) collisions. In this case the EDF body at $w < \varepsilon_1$ (w being the kinetic energy) decreases exponentially and the average electron energy is significantly less than ε_1. The direction of the differential fluxes in the EDF body depends on characteristics of the elastic scattering. In the atomic gases the energy fraction lost in one elastic collision equals $2m_e/M$ and does not depend on energy. In this case, for the power approximation of the transport free path $\lambda \propto v^{\kappa}$, the differential fluxes at $\kappa > 1$ are directed outward at large velocities (with respect to the average thermal velocity) and directed inward for the slow electrons. At $\kappa < 1$ the situation is inverse—the differential fluxes of the slow electrons flow outward and the fast ones flow inward.

If the EDF body is local, but the elastic energy losses remain small (such a situation being possible in heavy atomic gases), the differential fluxes of slow ($w \ll \varepsilon_1$) electrons are directed inward and the electrons with $w \lesssim \varepsilon_1$ flow towards the tube axis. In any case, in the EDF tail, with the exception of the wall vicinity, these fluxes remain directed inward.

At low pressure, when $\lambda < R$, the EDF of the outgoing particles becomes strongly anisotropic [15,16]. Ambipolar electron flux is caused in this case by scattering of the fast electrons into the loss cone [15]. At $\varepsilon < e\Phi_w$ the EDF remains almost isotropic and ε dependent. Below this case shall not be considered in detail.

III. KINETIC EQUATION IN VARIABLES OF TOTAL ENERGY AND POSITION

The problem for a standard cylindrical positive column shall be formulated in (ε, r) variables. The pressure is con-

sidered to be not too low in order to use the two-term expansion for the EDF:

$$f(\varepsilon,r)=f_0(\varepsilon,r)+\vec{f}_1(\varepsilon,r)\cdot\frac{\vec{v}}{v}, \tag{2}$$

with $f_0 \gg f_1$. Neglecting the energy loss by elastic collisions, the kinetic equation for the isotropic part of the EDF $f_0(\varepsilon,r)$ in these variables is of the form of

$$\frac{\partial}{\partial\varepsilon}\nu\frac{(eE_z\lambda)^2\nu}{3}\frac{\partial f_0}{\partial\varepsilon}+\frac{1}{r}\frac{\partial}{\partial r}r\nu\frac{\lambda^2\nu}{3}\frac{\partial f_0}{\partial r}$$
$$=\sum_k\left[\nu_k^*(v)\nu f_0(\varepsilon,r)-\sqrt{v^2+2\varepsilon_k/m}\right.$$
$$\left.\times\nu_k^*(\sqrt{v^2+2\varepsilon_k/m})f_0(\varepsilon+\varepsilon_k,r)\right]-I(\varepsilon,r), \tag{3}$$

where λ, ν, and ν_k^* are, respectively, the velocity-dependent momentum transfer mean free path, momentum transfer collision frequency, and inelastic collision frequency with excitation of the kth atomic level with energy ε_k from the ground state. The heating longitudinal electric field E_z is considered as radially homogenous. The last term on the right-hand side (RHS) of Eq. (3) corresponds to the slow electrons which are produced by ionization in the plasma. The electron-electron collision term may be neglected in this equation for relatively low electron densities. It is to be noted that a strict condition for such an expansion (2) is $R \gg \lambda$, where R is the tube radius. But since in real discharge conditions the relaxation of momentum practically always occurs considerably faster than energy relaxation, the expansion of the type of Eq. (2) and the equation for the main part of the EDF, which is formally identical to Eq. (3), can be derived also in the collisionless limit $R<\lambda$ [15]. An advantage of this form for the kinetic equation is given by the fact that it is reduced to the standard two-dimensional diffusion or thermal conductivity equation with sources and sinks in the RHS. The factors $D_\varepsilon(\nu(\varepsilon,r))=(eE_z\lambda)^2\nu\nu/3$ and $D_r(\nu(\varepsilon,r))=\lambda^2\nu\nu/3$ can be interpreted as variable effective diffusion coefficients for energy and ordinary space. Accordingly, in this analysis one can use methods, models, and analogs from these well-developed fields. The additional advantage of the chosen notation is due to the simplicity of the expressions for the axial and radial EDF anisotropy:

$$f_{1z}=-\frac{eE_z\nu}{\nu}\frac{\partial f_0}{\partial\varepsilon},$$

$$f_{1r}=-\frac{\nu}{\nu}\frac{\partial f_0}{\partial r}. \tag{4}$$

The standard normalization condition shall be assumed:

$$4\pi\int_{e\Phi(r)}^{\infty}\sqrt{\varepsilon-e\Phi(r)}f_0(\varepsilon,r)d\varepsilon=n(r). \tag{5}$$

The total spatial electron flux is

$$\vec{\Gamma}(r)=\frac{4\pi}{3}\sqrt{\frac{2}{m}}\int_{e\Phi(r)}^{\infty}[\varepsilon-e\Phi(r)]\vec{f}_1(\varepsilon,r)d\varepsilon. \tag{6}$$

The integrand in Eq. (6) corresponds to the differential spatial flux.

In order to obtain a simple classification, the dimensionless variables $\tilde{r}=r/R$ and $\tilde{\varepsilon}=\varepsilon/\varepsilon_1$ shall be introduced, where ε_1 is the energy of the first excitation level (in atomic gases it is of the order of all other excitation energies and of the ionization energy). The last two terms on the RHS of Eq. (3) play a role only at comparatively low energies (region I in Fig. 1). Neglecting them, one obtains the following equation:

$$A^2\frac{1}{\tilde{r}}\frac{\partial}{\partial\tilde{r}}\tilde{r}\tilde{D}\frac{\partial f_0}{\partial\tilde{r}}+\frac{\partial}{\partial\tilde{\varepsilon}}\tilde{D}_\varepsilon\frac{\partial f_0}{\partial\tilde{\varepsilon}}=\tilde{\nu}^*f_0. \tag{7}$$

There $A=\varepsilon_1/(eE_zR)$, the dimensionless diffusion coefficients $\tilde{D}(\tilde{\varepsilon},\tilde{r})=\tilde{D}_\varepsilon(\tilde{\varepsilon},\tilde{r})=\lambda\nu^2/(\lambda_1\nu_1^2)$ are of the order of unity, and $\tilde{\nu}^*(\tilde{\varepsilon},\tilde{r})=3\nu\varepsilon_1^2/(e^2E_z^2\nu_1^2\lambda_1)\Sigma_k\nu_k^*$ is the summary excitation frequency in units of inverse electron energy diffusion time over ε_1 [$\nu_1=\sqrt{2\varepsilon_1/m}$, $\lambda_1=\lambda(v=v_1)$]. For the special case of the dc positive column, the dimensionless coefficients of the spatial diffusion $\tilde{D}$ and of the energy diffusion $\tilde{D}_\varepsilon$ coincide. But for practically all the other cases of interest, they are substantially different, first of all, due to spatial dependence in the energy input. It follows that in addition to the $\tilde{r}$ dependence which results from the dependence of $\tilde{D}_\varepsilon$ on $\nu(\tilde{\varepsilon},\tilde{r})$, in the general case this coefficient contains also some additional $\tilde{r}$-dependent factor.

In the energy range in which the vast majority of the inelastic collisions occurs, the parameter ν^*, as a rule, considerably exceeds unity. In the opposite case the division of the EDF into a body and fast-decaying tail becomes senseless, since the EDF at $\varepsilon\to1$ decreases with the same energy scale as at lower energy. At $\nu^*\gg1$ in the absence of the elastic energy losses an electron gains energy $\geqslant\varepsilon_1$ in the energy diffusion process during a time $\sim(\varepsilon/D)$, and the energy relaxation length, in which an electron gains and loses energy of the order of ε_1, equals $\lambda_\varepsilon=\varepsilon_1/(eE_z)$. The parameter A corresponds to the ratio of this length to the tube radius. If $A\gg1$, the body of the EDF at $\tilde{\varepsilon}\leqslant1$ is nonlocal. For nonlocality of the EDF tail the more strict condition $\overline{\nu^*}\ll A^2$ is necessary. It means that cases are possible when the EDF body is close to nonlocal, but at the tail the deviations are considerable or even the EDF is practically local. Since $\overline{\nu^*}$ increases with energy, such a situation is characteristic for higher energies. The discussion in the Secs. V–VII shall be restricted mainly to the case when $A\gg1$. It means that the body of the EDF at $\tilde{\varepsilon}\leqslant1$ is nonlocal as it holds for the calculations [7,8]. But for $\tilde{\nu}^*$ and $\delta\tilde{\Omega}$ [see below, Eq. (15)], both limiting cases will be considered.

The radial potential profile $\tilde{\Phi}(\tilde{r})$ and the longitudinal field E_z in the rigorous formulation are to be found self-consistently, taking into account the ion evolution, the quasineutrality condition, and the discharge stationarity [15]. But for simplicity they shall be considered as *ad hoc* prescribed characteristics. Since the ion motion is characterized by a considerably longer time scale than the electron processes, it means that, if even at given values of E_z, $\tilde{\Phi}(\tilde{r})$, and $\tilde{\Phi}_w$ the above-mentioned conditions are not fulfilled, one can consider the problem as a description of the starting

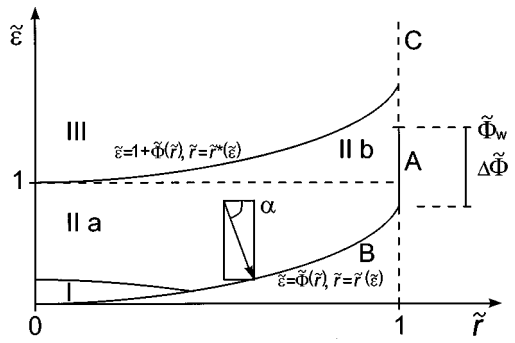

FIG. 1. Integration domain Eq. (7) in the $(\tilde{r},\tilde{\varepsilon})$ plane. The dimensionless variables $\tilde{\varepsilon}$ and $\tilde{r}$ are defined in the text.

phase of some slow ion relaxation process to the real stationary state. In any case a more or less realistic radial potential profile consists of the plasma part, which is convex in the vicinity of the discharge axis, steepening towards the wall, and of a potential fall in the sheath at the tube surface. At not too high pressures its thickness, which is of the order of the local Debye radius, is small with respect both to the tube radius and to the mean free path. Accordingly, this part of the potential profile is considered traditionally as infinitely thin, but its potential fall $\Delta\tilde{\Phi}$ cannot be neglected in order to get the proper boundary conditions at the tube wall. It follows that the potential profile used in [7], where the sheath fall was neglected, is in some sense artificial.

IV. BOUNDARY CONDITIONS

The boundary condition at the tube axis is trivial:

$$\left.\frac{\partial f_0}{\partial\tilde{r}}\right|_{\tilde{r}=0}=0. \tag{8}$$

The same condition is valid also at $\tilde{\varepsilon}<\tilde{\Phi}_w$, $\tilde{r}=1$ (part A in Fig. 1), which corresponds to the sheath. The condition at the zero-velocity boundary (part B in Fig. 1) demands the absence of influx of slow electrons:

$$\tilde{\Gamma}_{\perp,B}=(-\tilde{\Gamma}_\varepsilon\sin\alpha+\tilde{\Gamma}_r\cos\alpha)\big|_B = \left(\tilde{D}_\varepsilon\frac{\partial f}{\partial\tilde{\varepsilon}}\sin\alpha-A^2\tilde{D}\frac{\partial f}{\partial\tilde{r}}\cos\alpha\right)\Bigg|_B=0, \tag{9}$$

where $\cot\alpha=d\tilde{\Phi}/d\tilde{r}$. The possible singularity of the EDF at this boundary, which arises if $\tilde{D},\tilde{D}_\varepsilon\to 0$ here, was discussed in [17].

More complicated is the boundary condition at $\tilde{r}=1$, $\tilde{\varepsilon}>\tilde{\Phi}_w$, which describes the possible escape of fast electrons to the tube wall. A discussion of this problem can be found in [3,8]. Since the escape of the fast electrons corresponds to an additional very effective loss mechanism, the EDF at $\tilde{\varepsilon}>\tilde{\Phi}_w$ decreases very fast. It follows that for electrons which play some noticeable role in plasma processes the loss cone $\delta\Omega(\tilde{\varepsilon})$ (i.e., the range of incidence angles in which electrons reach the wall) is relatively small. According to this, in [3] a simple expression was proposed for this boundary condition which is valid at

$$\delta\Omega(\tilde{\varepsilon})=2\pi[1-\cos\chi^*(\tilde{\varepsilon})]\ll 4\pi. \tag{10}$$

The limiting angle $\chi^*(\tilde{\varepsilon})$ which corresponds to the transition from escaping to trapped electrons can be estimated as

$$\cos\chi^*(\tilde{\varepsilon})=\sqrt{\frac{\Delta\tilde{\Phi}}{\tilde{\varepsilon}-\tilde{\Phi}_w+\Delta\tilde{\Phi}}}. \tag{11}$$

The anisotropy of the EDF in the immediate vicinity of the wall cannot be described in the framework of the two-term approximation (2). This is due to the fact that the ingoing particles (at $-1<\cos\theta<-\cos\chi^*$) are absent here and the antiloss cone is empty. Correspondingly, the higher components in the expansion of the EDF over spherical harmonics are comparable to the first-order one, but are small with respect to its isotropic part f_0 (of the order of $f_0\delta\Omega/4\pi$). If we neglect them, the differential flux to the wall is equal to $\delta\Omega(\tilde{\varepsilon})f_0$. On the other hand, at large distance from the wall (with respect to λ, but small compared to the tube radius R), the two-term approximation (2) is valid, and since the differential flux is conserved at such a small distance, one has the necessary boundary condition on C (Fig. 1):

$$\frac{4\pi}{3}f_1=\delta\Omega f_0=-\frac{4\pi}{3}\tilde{\lambda}\frac{\partial f_0}{\partial\tilde{r}}, \tag{12}$$

where the dimensionless $\tilde{\lambda}=\lambda/R$. A rigorous derivation of the boundary condition for the case $\delta\Omega\sim 4\pi$ demands a complicated solution of the full kinetic equation with accounting for the angular dependence of the scattering cross section. It cannot be performed in the two-term approximation. To our knowledge, it was not performed up to now. But it is well known that for the special case when the sheath is absent ($\delta\Omega=2\pi$) the result of the two-term approximation, which is identical to Eq. (12) for this case, as well as for $\delta\Omega\ll 4\pi$, coincides with surprising accuracy with the accurate kinetic result for the isotropic electron-atom scattering. A rigorous calculation then gives a coefficient for the ratio of f_0 to f_1 in Eq. (12) of 0.7104 instead of the 2/3 in the two-term approximation [18]. The expression for arbitrary $\delta\Omega$ was obtained in the two-term approximation in [8]. A nonzero reflection coefficient ζ was also accounted for:

$$-\tilde{\lambda}\frac{\partial f_0}{\partial\tilde{r}}=\frac{3}{2}f_0\frac{(1-\cos^2\chi^*)(1-\zeta)}{1+\zeta+\cos^2\chi^*(1-\zeta)}\Bigg|_C. \tag{13}$$

From Eq. (12) at $\zeta\neq 0$ it follows that

$$-\tilde{\lambda}\frac{\partial f_0}{\partial\tilde{r}}=\frac{3}{2}f_0(1-\cos\chi^*)(1-\zeta)\Big|_C. \tag{14}$$

Equations (13) and (14) coincide at small $\delta\Omega$; at $\zeta=0$, they coincide also at $\cos\chi^*=0$. The deviation of their ratio from unity does not exceed 25% at almost all values of $\cos\chi^*$. This ratio is maximal (up to 2) in the rather exotic case $\zeta\to 1$, $\cos\chi^*\sim 1$. Having in mind that both these expressions (13) and (14) are not too rigorous, the second one (14) shall be preferred with view to its simplicity:

$$-\frac{\partial f_0}{\partial \tilde{r}}=\frac{3(1-\zeta)}{2\tilde{\lambda}[\nu(\tilde{\varepsilon},\tilde{r})]}[1-\cos\chi^*(\tilde{\varepsilon})]f_0=\delta\tilde{\Omega}(\tilde{\varepsilon})f_0\Big|_C, \tag{15}$$

where the effective loss cone $\delta\tilde{\Omega}$ is defined. In the calculations below, the assumption $\zeta=0$ is used.

V. EDF BODY IN THE NONLOCAL CASE

For this case Eq. (7) reduces to the ordinary Laplace-type equation. For the nonlocal case $A\gg 1$, $\tilde{\varepsilon}<1$, the isotropic part of the EDF $f_0(\tilde{\varepsilon},\tilde{r})$ is mainly $\tilde{\varepsilon}$ dependent, and small $\tilde{r}$-dependent terms $f_0^{(1)}(\tilde{\varepsilon},\tilde{r})$ can be calculated by an iterative procedure [2]. The problem coincides with the problem of thermal conductivity in a long inhomogeneous rod of variable cross section with insulated side boundaries. Substituting the expansion

$$f_0(\tilde{\varepsilon},\tilde{r})=f_0^{(0)}(\tilde{\varepsilon})+A^{-2}f_0^{(1)}(\tilde{\varepsilon},\tilde{r})+\cdots \tag{16}$$

into Eq. (7), integrating it in $\tilde{r}$ over the available cross section $0<\tilde{r}<\hat{r}(\tilde{\varepsilon})$, and using the boundary conditions (8) and (9), one obtains, for $f_0^{(0)}$ at $\tilde{\varepsilon}<\min(1,\tilde{\Phi}_\omega)$ (region IIa in Fig. 1),

$$\frac{d}{d\tilde{\varepsilon}}\overline{D_\varepsilon}(\tilde{\varepsilon})\frac{df_0^{(0)}}{d\tilde{\varepsilon}}=0, \tag{17}$$

where the spatially averaged energy diffusion coefficient is equal to

$$\overline{D_\varepsilon}(\tilde{\varepsilon})=\int_0^{\tilde{r}(\tilde{\varepsilon})}\tilde{r}\tilde{D}_\varepsilon(\tilde{\varepsilon},\tilde{r})d\tilde{r}, \tag{18}$$

with $\tilde{r}(\tilde{\varepsilon})$ satisfying $\tilde{\Phi}(\tilde{r}(\tilde{\varepsilon}))=\tilde{\varepsilon}$ at $\tilde{\varepsilon}<(\tilde{\Phi}_\omega-\Delta\tilde{\Phi})$ or $\tilde{r}(\tilde{\varepsilon})=1$ in the opposite case. For $f_0^{(1)}(\tilde{\varepsilon},\tilde{r})$ one has the equation

$$-\frac{A^2}{\tilde{r}}\frac{\partial}{\partial\tilde{r}}\tilde{r}\tilde{D}(\tilde{\varepsilon},\tilde{r})\frac{\partial f_0^{(1)}}{\partial\tilde{r}}=\frac{\partial}{\partial\tilde{\varepsilon}}\tilde{D}_\varepsilon(\tilde{\varepsilon},\tilde{r})\frac{\partial f_0^{(0)}}{\partial\tilde{\varepsilon}}. \tag{19}$$

Integrating Eq. (19) over $\tilde{r}$ once,

$$\Gamma_r(\tilde{\varepsilon},\tilde{r})=-\tilde{D}\frac{\partial f_0^{(1)}}{\partial\tilde{r}}=\frac{1}{A^2}\frac{1}{\tilde{r}}\frac{\partial}{\partial\tilde{\varepsilon}}\frac{\partial f_0^{(0)}}{\partial\tilde{\varepsilon}}\int_0^{\tilde{r}}\tilde{r}'\tilde{D}_\varepsilon(\tilde{\varepsilon},\tilde{r}')d\tilde{r}' \tag{20}$$

is obtained. According to Eq. (4), this expression is proportional to the radial differential flux of interest here. From Eqs. (17) and (18) one has

$$\frac{\partial f_0^{(0)}}{\partial\tilde{\varepsilon}}=-\frac{\Gamma}{\displaystyle\int_0^{\tilde{r}(\tilde{\varepsilon})}\tilde{r}'\tilde{D}_\varepsilon(\tilde{\varepsilon},\tilde{r}')d\tilde{r}'}, \tag{21}$$

where $\Gamma>0$ denotes the total (conserved) flux in energy direction integrated over the available cross section. This flux is directed upward in ε and corresponds to the electron energy gain starting from slow-injected electrons which appear as a result of excitation or ionization processes up to the high-energy region where they are lost due to inelastic collisions [at $\tilde{r}<\tilde{r}^*(\tilde{\varepsilon})$; region III Fig. 1] or due to the escape to the wall at $\tilde{\varepsilon}>\tilde{\Phi}_w$. The solution (21) is proportional to Γ; it can be considered as the normalization factor. Substituting Eq. (21) into Eq. (20), one obtains the desired result:

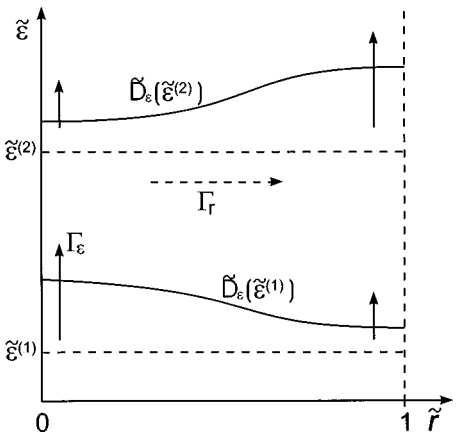

FIG. 2. Formation scheme of radial fluxes due to the spatially inhomogeneous energy diffusion.

$$\Gamma_r(\tilde{\varepsilon},\tilde{r})=-\frac{1}{A^2}\frac{\Gamma}{\tilde{r}}\frac{\partial}{\partial\tilde{\varepsilon}}\left(\frac{\displaystyle\int_0^{\tilde{r}}\tilde{r}'\tilde{D}_\varepsilon(\tilde{\varepsilon},\tilde{r}')d\tilde{r}'}{\displaystyle\int_0^{\tilde{r}(\tilde{\varepsilon})}\tilde{r}''\tilde{D}_\varepsilon(\tilde{\varepsilon},\tilde{r}'')d\tilde{r}''}\right). \tag{22}$$

It is clearly seen from Eq. (22) that there are two totally different physical mechanisms which result in the formation of radial fluxes. The first is connected to the expansion of the available cross section with energy and is described by the energy dependence of the denominator in Eq. (22). It exists also for arbitrary velocity- and $\tilde{r}$-independent $\tilde{D}_\varepsilon$ and always results in a flux directed outward since the expansion of the available area results in a denominator growing with $\tilde{\varepsilon}$ in Eq. (22). The second mechanism is determined by the combined action of the spatial and energy dependence of $\tilde{D}_\varepsilon$ and can be, in principle, directed both inward to and outward from the plasma bulk. It can be understood using the above-mentioned analogy with the thermal conductivity. If in a long thin inhomogeneous cylindric rod with insulated sides the arbitrary ''temperature'' profile at $\tilde{\varepsilon}=0.1$ is given, in its main part the temperature is (almost) $\tilde{r}$ independent. The flux density in energy $\Gamma_\varepsilon(\tilde{\varepsilon},\tilde{r})=-\tilde{D}_\varepsilon\partial f_0^{(0)}/\partial\tilde{\varepsilon}$ at a given $\tilde{\varepsilon}$ simply reproduces $\tilde{r}$-dependence of $\tilde{D}_\varepsilon(\tilde{\varepsilon},\tilde{r})$, the total flux in energy $\Gamma=\int_0^1\tilde{r}\Gamma_\varepsilon(\tilde{\varepsilon},\tilde{r})d\tilde{r}$ being energy independent. If the dependence $\tilde{D}_\varepsilon(\tilde{\varepsilon},\tilde{r})$ can be factorized into separate $\tilde{\varepsilon}$- and $\tilde{r}$-dependent factors, the radial flux Γ_r is absent. If such a factorization of $\tilde{D}_\varepsilon(\tilde{\varepsilon},\tilde{r})$ is impossible, the different portions of this flux Γ are transported at different $\tilde{\varepsilon}$ by different parts of the rod cross section and radial fluxes arise. For the special case of the dc positive column in inert gases, $\tilde{D}_\varepsilon$ increases with the kinetic energy and the dependence $\tilde{D}_\varepsilon(\tilde{r})$ at a given $\tilde{\varepsilon}$, which results from the space-charge field $\tilde{\Phi}(\tilde{r})$, levels out at higher $\tilde{\varepsilon}$. It means that this mechanism also

leads to a differential flux $\Gamma_r(\tilde{\varepsilon})$ directed outward, even when the available cross section is constant. This situation is sketched in Fig. 2.

In the calculations of [7] for neon, the authors used $\tilde{\Phi}_w = 1.2$, $\Delta\tilde{\Phi} = 0$, and $A = 4.5$. In accordance with this, the EDF body was practically nonlocal, and thus the EDF's in different points of the tube cross section at a given total energy ε coincided. The differential fluxes at $\varepsilon < \varepsilon_1$ were directed outward. The same results were obtained for helium in [8], in spite of the fact that the parameter A was relatively small (of the order of 0.5).

VI. EDF TAIL

At $A \gg 1$ it starts at $\tilde{\varepsilon} > \min(1, \tilde{\Phi}_w)$, when the loss mechanisms start to play a role. In this region of phase space, as a rule, for a dc positive column the coefficients $\tilde{D}$ and $\tilde{D}_\varepsilon$ can be treated as constants of the order of unity, $\tilde{r}(\tilde{\varepsilon}) = 1$, and one can simplify Eq. (7) to

$$A^2 \frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} \tilde{r} \frac{\partial f_0}{\partial \tilde{r}} + \frac{\partial^2 f_0}{\partial \tilde{\varepsilon}^2} = \tilde{\nu}^* f_0, \tag{23}$$

with the boundary conditions

$$f_0|_{\tilde{\varepsilon} \to \infty} = 0,$$

$$-\frac{\partial f_0}{\partial \tilde{r}} = \delta\tilde{\Omega}(\tilde{\varepsilon}) f_0 \Big|_{\tilde{r}=1}, \tag{24}$$

where $\delta\tilde{\Omega}(\tilde{\varepsilon}) = 0$ at $\varepsilon < e\Phi_w$. If $\tilde{\nu}^*(\tilde{\varepsilon}, \tilde{r}) \ll A^2$, $\delta\tilde{\Omega}(\tilde{\varepsilon}) \ll 1$, the inelastic collisions and escapes to the wall are rare with respect to radial diffusion, the EDF tail is also nonlocal in the sense that the main part of the EDF $f_0(\tilde{\varepsilon}, \tilde{r})$ depends only on $\tilde{\varepsilon}$. Using the same kind of expansion as Eq. (16), one obtains, for $f_0^{(0)}(\tilde{\varepsilon})$,

$$\frac{d^2 f_0^{(0)}(\tilde{\varepsilon})}{d\tilde{\varepsilon}^2} = \overline{\nu^*}(\tilde{\varepsilon}) f_0^{(0)}(\tilde{\varepsilon}), \tag{25}$$

where the average loss rate is

$$\overline{\nu^*}(\tilde{\varepsilon}) = \frac{2}{\tilde{r}^2(\tilde{\varepsilon})} \int_0^{\tilde{r}^*(\tilde{\varepsilon})} \tilde{\nu}^*(\tilde{\varepsilon}, \tilde{r}) \tilde{r}\, d\tilde{r} + 2A^2 \delta\tilde{\Omega}(\tilde{\varepsilon}). \tag{26}$$

The solution of Eqs. (25) and (26) can be found analytically for some special cases of $\overline{\nu^*}(\tilde{\varepsilon})$ dependence or calculated numerically by standard methods. The expression for the differential radial flux can be obtained from Eq. (23) in a manner similar to Eqs. (20)–(22). Integrating Eq. (23) over $\tilde{r}$, one obtains

$$\Gamma_r(\tilde{\varepsilon}, \tilde{r}) = -A^2 \frac{\partial f_0^{(1)}}{\partial \tilde{r}} = \frac{1}{\tilde{r}} \int_0^{\tilde{r}} \left[\frac{d^2 f_0^{(0)}}{d\tilde{\varepsilon}^2} - \tilde{\nu}^*(\tilde{\varepsilon}, \tilde{r}) f_0^{(0)} \right] \tilde{r}'\, d\tilde{r}'. \tag{27}$$

Substituting Eqs. (25) and (26) yields

$$\Gamma_r(\tilde{\varepsilon}, \tilde{r}) = f_0^{(0)}(\tilde{\varepsilon}) \left\{ A^2 \tilde{r}\, \delta\tilde{\Omega}(\tilde{\varepsilon}) - \left[\frac{1}{\tilde{r}} \int_0^{\tilde{r}} \tilde{\nu}^* \tilde{r}'\, d\tilde{r}' - \frac{\tilde{r}}{\tilde{r}^2(\tilde{\varepsilon})} \int_0^{\tilde{r}^*(\tilde{\varepsilon})} \tilde{\nu}^* \tilde{r}'\, d\tilde{r}' \right] \right\}. \tag{28}$$

The first term on the RHS corresponds to the positive differential flux (directed outward) due to the wall losses. The second one is practically always (at nonexotic dependences of $\tilde{\nu}^*$ on the electron velocity) negative and corresponds to the flux directed inward which is generated by energy losses in the inelastic collisions, which occur mainly in the tube center.

The more complicated situation corresponds to the case when either $\delta\tilde{\Omega} \sim 1$ or $\tilde{\nu}^* \geqslant A^2$. These inequalities mean that due to wall losses and/or to the inelastic collisions the spatial dependence of the EDF becomes significant. The strong inequalities

$$\delta\tilde{\Omega} \gg 1, \quad \tilde{\nu}^* \gg A^2 \tag{29}$$

mean that the EDF is small in the vicinity of the line C and in the inelastic region III in Fig. 1, and in a first approximation zero-boundary conditions can be imposed at $\tilde{r} = \tilde{r}^*(\tilde{\varepsilon})$, 1. Such a case corresponds to the calculations of [7,8], and for simplicity this approximation shall be used. This case formally corresponds to the case with a nonlocal EDF and local tail. Real situations, however, are far more complicated. The EDF tail in III (Fig. 1) is local in the sense that its exponential decay is determined by the local parameters. In the region IIb inelastic collisions are absent and their influence results in intense inward-directed differential fluxes and to outward-directed ones near the wall. The solution in the region IIb satisfies the Laplace equation [2].

$$A^2 \frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} \tilde{r} \frac{\partial f_0}{\partial \tilde{r}} + \frac{\partial^2 f_0}{\partial \tilde{\varepsilon}^2} = 0. \tag{30}$$

If the dependence $\tilde{r}^*(\tilde{\varepsilon})$ is smooth enough, the quasiclassical solution of Eq. (30) in the region IIb can be proposed [5] in the form

$$f_0(\tilde{\varepsilon}, \tilde{r}) \sim \exp\left(-\int_1^{\tilde{\varepsilon}} K(\tilde{\varepsilon}')\, d\tilde{\varepsilon}' \right) \Psi(\tilde{r}), \tag{31}$$

where $\Psi(\tilde{r})$ depends on $\tilde{\varepsilon}$ only as on a parameter (i.e., Ψ varies slowly with $\tilde{\varepsilon}$) and satisfies the equation

$$A^2 \frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} \tilde{r} \frac{\partial \Psi}{\partial \tilde{r}} + K^2 \Psi = 0, \tag{32}$$

with boundary conditions $\Psi(\tilde{r}) = 0$ at $\tilde{r} = \tilde{r}^*(\tilde{\varepsilon})$, 1. The solution of Eq. (32) can be expressed by Bessel and Neumann functions of the zeroth order. In the plane geometry [or at $\tilde{r}(\tilde{\varepsilon} \to 1)$], one can simply write

$$\Psi(\tilde{r}) = \sin\left[\frac{\pi[\tilde{r} - \tilde{r}^*(\tilde{\varepsilon})]}{1 - \tilde{r}^*(\tilde{\varepsilon})} \right],$$

$$K(\tilde{\varepsilon}) = \frac{A\pi}{1 - \tilde{r}^*(\tilde{\varepsilon})}. \tag{33}$$

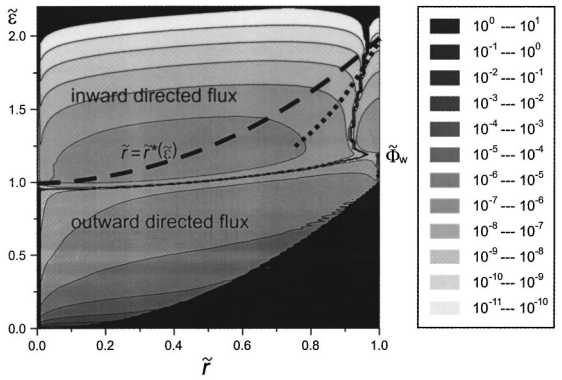

FIG. 3. Relative values of radial differential fluxes in a parabolic potential (as simplified example) profile: $\tilde{\Phi}(\tilde{r})=\tilde{r}^2$; $\tilde{\Phi}_w=1.2$ in Ne. The dashed line corresponds to the boundary of the inelastic region $\tilde{r}=\tilde{r}^*(\tilde{\varepsilon})$; $\tilde{\Phi}(\tilde{r}^*)=\tilde{\varepsilon}-1$. The positive fluxes in region IIa and the negative ones in region III can clearly be seen. The dotted line corresponds to approximation (34) for the boundary $\Gamma_r(\tilde{\varepsilon},\tilde{r})=0$, at which the outward-directed energy flux to the tube wall is equal to the inward-directed one.

The solution (31) for the case (29) decrease with energy $\tilde{\varepsilon}$ in the characteristic scale $\Delta\tilde{\varepsilon}\sim(A\pi)^{-1}$. The differential radial fluxes in this region IIa, according to Eqs. (33) and (4), change sign. The inequalities (29) correspond to conditions of the calculation [7] ($A\approx 4.5$, $\tilde{\nu}^*\approx 170$ at $\varepsilon=20$ eV and $\tilde{\nu}^*\approx 400$ at $\varepsilon=30$ eV). The authors of [7] have not found this sign change, possibly due to the artificial boundary conditions which were used in their calculations. The results of the calculations of f_{1r} for the same case with the corrected boundary condition (13) ($\tilde{\Phi}_w=1.2$) are presented in the Figs. 3 and 4.

As well as in [8], two lines, at which the differential fluxes change their sign, are clearly seen in Fig. 3. One of them corresponds to the boundary of the EDF body $\tilde{\varepsilon}\approx 1$. The reason for its slight deviation from $\tilde{\varepsilon}=1$ in the tube center in the calculations of [8] remains unclear. The second boundary at $\varepsilon>e\Phi_w$ can be attributed to the transition from the escape-dominated flux to the flux determined by the inelastic collisions. In the case of neon ([7], calculations here) the main part of this boundary lies in the region IIb. As is seen from Fig. 3, its position does not deviate too strongly from

$$\tilde{r}(\tilde{\varepsilon})=\frac{\tilde{r}^*(\tilde{\varepsilon})+1}{2}, \tag{34}$$

which follows from Eq. (33). In the case of helium [8] this whole boundary lies practically in the inelastic region III, and its behavior is significantly different. If for neon this boundary tends to be vertical, in helium it had a significant slope, so that at moderate E_z the domain of flux directed inward was restricted in the (ε,r) plane. This distinction can mainly be attributed to the different behavior of the elastic electron-atomic cross sections in helium and neon. In the case of neon in the whole energy interval investigated it is practically constant. But in helium it decreases fast with energy, and at some high energy the second of inequalities (29) is violated. In this case the EDF tail becomes nonlocal, and according Eq. (28), the flux, even at the tube axis, becomes

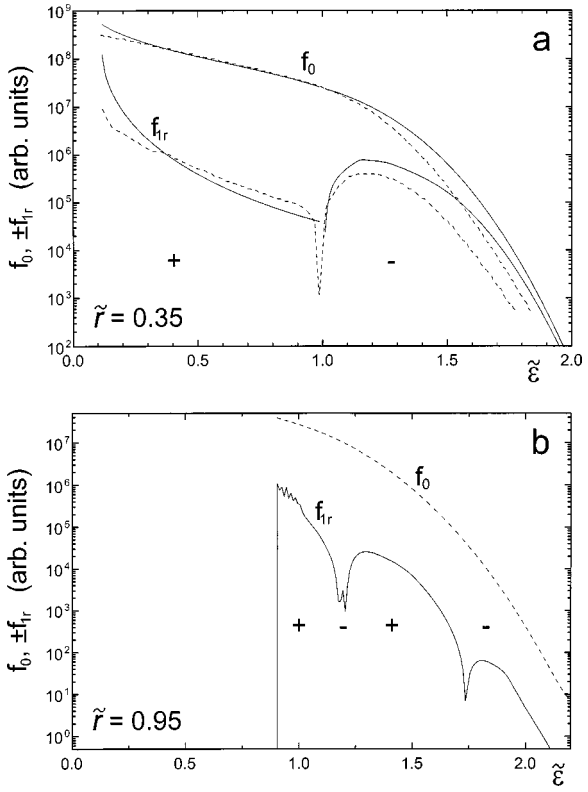

FIG. 4. (a) The results of [6] (dashed lines) and our results (solid lines) for Ne at $a=1.7$ cm, $p=100$ Pa, and $r=0.6$ cm. (b) The EDF near the wall ($r=16.2$ cm) according Eq. (7). Two flux reversals can be seen. One of them corresponds to the transition from the outward-directed flux of slow electrons in region IIa to the inward-directed one in region IIa, which is caused by inelastic collisions. The second reversal corresponds to the electron outflux to the tube walls.

positive. A rough estimate according to $\tilde{\nu}^*$ $(\varepsilon_{cr})=1$ gives for the critical energy of this transition $\varepsilon_{cr}\sim 100$ eV in the first of the cases investigated in [8] ($NR=1.2\times 10^{17}$ cm^{-2}, $E_z/N=3\times 10^{16}$ V cm^2). This value agrees with $\varepsilon_{cr}=80$ eV in the calculations. At higher pressure this transition shifts to considerably higher energies, in agreement with [8].

The isotropic (solid lines) and radially anisotropic (dashed lines) parts of the EDF in neon are presented in Fig. 4. It can be clearly seen that the EDF body is nonlocal, but for the tail the nonlocal approximations (21) and (25) results in too high values of f_0 [solid curve $3a$ in Fig. 4(a)]. It means that in the inelastic region III the radial fluxes from region IIb are important. In order to estimate their influence, in region III the diffusion in energy shall be neglected, assuming that the EDF f_0 here satisfies the simple equation

$$A^2\frac{1}{\tilde{r}}\frac{\partial}{\partial\tilde{r}}\tilde{r}\frac{\partial f_0}{\partial\tilde{r}}=\tilde{\nu}^*(\tilde{\varepsilon},\tilde{r})f_0. \tag{35}$$

The boundary condition $\partial f_0/\partial\tilde{r}|_{\tilde{r}}=0$ of the EDF symmetry can be replaced in this case $A^2\ll\tilde{\nu}^*$ by the condition of absorbing boundary at $\tilde{r}=\tilde{r}^*(\tilde{\varepsilon})$: $f_0\{[\tilde{r}-\tilde{r}^*(\tilde{\varepsilon})]\to-\infty\}=0$. At $\tilde{r}=\tilde{r}^*(\tilde{\varepsilon})$ the solution of Eq. (35) is to be matched (with its normal derivative) with the solution of Eq. (30).

The anisotropy of the EDF body is satisfactorily described by the nonlocal expression (22), but in the tail region, in

accordance with Eq. (29), the nonlocal calculations (27) and (28) lead to an overestimation of the anisotropy.

The situation at $\tilde{\varepsilon} > \tilde{\Phi}_w - \Delta\tilde{\Phi} + 1$, when the region IIb ends, is more complicated, and it shall not be discussed here in detail.

It follows that the mechanisms of the formation of the differential radial fluxes at the EDF tail result from the wall escape and from inelastic collisions, both these mechanisms being ineffective for the EDF body. On the other hand, the mechanisms, which are effective at the lower energies, are ineffective at the EDF tail.

A specific mechanism of the differential spatial fluxes formation occurs in rf and UHF discharges, when the energy input density is strongly inhomogeneous. This inhomogeneity can be connected with the skin effect in inductively coupled discharges, with increase of the rf field strength in the peripheric more rarefied plasma which follows from conservation of the rf current in the capacitively coupled discharges, with the focusing and absorption of the UHF wave in UHF discharges, etc. Practically, in all these situations it is observed that at low pressures $A^2 \gg \tilde{\nu}^*$, the maximum of luminosity (and most probably of the ionization rate) is positioned in the geometric center of the plasma vessel. It is consistent with the nonlocal scenario, when, due to extremely fast electron transport, the EDF depends on the full energy ε [19,20]. Starting with the potential profile $\Phi(\vec{r})$ with a minimum in the vessel center, one obtains a self-consistent scheme, since the ionization rate with the ε-dependent EDF in such a potential is also maximal in this point. And the ion equation with a symmetric distribution of the sources, which defines $\Phi(\vec{r})$, results in a symmetric profile $\Phi(\vec{r})$. But with the pressure increase the maximum of luminosity is shifted towards the maximum of energy input. On the other hand, in the local limit $1 \gg A^2$, the luminosity maximum is to be situated in the place, where the energy input (for example, the rf field strength) is maximal. But in the intermediate situations described above the situation is more complicated, and the point of maximal luminosity can be situated rather far both from the geometrical center of the discharge chamber and from the point of maximum energy input.

The case shall be considered when the EDF tail is local, and the EDF body is nonlocal ($1 \ll A^2 \ll \tilde{\nu}^*$), and the diffusion coefficient in energy is strongly peaked in some point $\tilde{r} = \tilde{r}_0 \sim 1$ far from the geometrical center:

$$\tilde{D}_\varepsilon = \delta(\tilde{r} - \tilde{r}_0). \tag{36}$$

The solution of Eq. (30) in the region IIb decays exponentially with $\tilde{\varepsilon}$ with characteristic scale of the order of A^{-1}. It means that the luminosity intensity (which shall be considered for simplicity as proportional to the total excitation rate) exponentially decreases with $\tilde{r}$ at $\tilde{r} > \tilde{r}_1$, which is given Eq. by (37). On the other hand, the luminosity at small $\tilde{r}$ first increases with $\tilde{r}$. This growth is caused by two reasons. After all, since the diffusion in energy occurs only at $\tilde{r} = \tilde{r}_0$, the differential spatial flux $\Gamma_r(\tilde{\varepsilon})$ into the absorbing region III (see Fig. 3) at $(\tilde{\varepsilon} - 1) \ll A^{-1}$ is inversely proportional to the distance between the position of source of the fast particles $\tilde{r}_0$ and the absorption point $\tilde{r} = \tilde{r}^*(\tilde{\varepsilon})$, which decrease with $\tilde{\varepsilon}$. The second reason follows from the fact that the excitation rate (per unit length) is proportional to $d\tilde{\Phi}/d\tilde{r}$ at the absorption point, which increases with $\tilde{\varepsilon}$. It follows that the maximal excitation rate occurs at the position $\tilde{r} \sim \tilde{r}_1$:

$$\tilde{\Phi}(\tilde{r}_1) \sim A^{-1}. \tag{37}$$

VII. EDF BODY IN THE LOCAL CASE

At higher pressures the value of A becomes less than unity, and both the EDF body and tail become local. In this case the fluid approach for calculation of the particles fluxes is formally valid, but the analysis in terms of the differential fluxes allows to obtain deeper insight in the underlying physics. The radial differential fluxes can be calculated in a more traditional way. In the zeroth approximation the isotropic part of the EDF satisfies

$$\frac{1}{v}\frac{d}{dw}\left(D_\varepsilon v \frac{df_0^{(0)}}{dw} + V_\varepsilon v f_0^{(0)} \right) = \nu^*(v) f_0^{(0)}, \tag{38}$$

where the energy diffusion coefficient $D_\varepsilon(w)$ and the rate of energy loss V_ε are given by

$$D_\varepsilon = (eE\lambda)^2 \nu/3, \quad V_\varepsilon = \delta\nu w, \quad \delta = 2m_e/M_a, \quad w < wv^2/2. \tag{39}$$

In molecular gases the energy losses by excitation of rotational and vibrational levels can usually be treated via quasi-elastic collisions. This means they can be described by introducing an energy-dependent parameter $\delta(w)$ [21]. The radial fluxes in the tail in this case can be found according to the previous section. With the exception of a small wall vicinity, they are directed inward.

In the EDF body $w < \varepsilon_1$, the situation is different depending on the mechanisms of the energy and momentum relaxation. If the energy relaxation is determined by the energy loss in the inelastic collisions, the second term on the LHS of Eq. (38) can be neglected and the EDF body in the ''absorbing wall'' approximation [6] is given by

$$f_0^{(0)}(w,r) = Bn(r) \int_w^{\varepsilon_1} \frac{dw'}{v' D_\varepsilon(w')} = Bn(r) \int_v^{\sqrt{2\varepsilon_1/m_e}} \frac{dv'\nu(v')}{(v')^2}. \tag{40}$$

The corresponding anisotropic part equals

$$f_{1r} = -\frac{v}{\nu}\left(\frac{\partial f_0^{(0)}}{\partial r}\right)_v - \frac{eE_r}{m\nu}\left(\frac{\partial f_0^{(0)}}{\partial v}\right)_r = -B\frac{v}{\nu}\int_v^{\sqrt{\varepsilon_1/m_e}} \frac{\nu(v')dv'}{(v')^2}\frac{dn}{dr} + Bn\frac{eE_r}{mv^2}. \tag{41}$$

The first term on the RHS is directed outward, and the second one corresponds to the inward flux. Since the average electron energy in this case is of the order of ε_1 [6], the electric field can be estimated as

$$E_r \sim \frac{\varepsilon_1}{eR}. \tag{42}$$

It means that both terms on the RHS of Eq. (41) are of the same order $\sim(\lambda/R)f_0^{(0)}$, which corresponds to free electron diffusion [22]. The standard ambipolar diffusion corresponds to the situation when these terms practically cancel each other. It means that from the absence of net electron current (more precisely, from the fact that it equals to the small ion one) follows the absence of the differential fluxes, too. This condition cannot be satisfied for a non-Maxwellian EDF. The electric field which corresponds to zero net current cannot compensate the differential fluxes at arbitrary energy. At high energies $w\sim\varepsilon_1$ the second term in Eq. (41) (the flux directed inward) dominates. At energies below the average energy of the distribution, the differential flux is directed outward.

This situation can be met in practice. For example, in the conditions of [8], which corresponded to transition to the local EDF, the contribution of the elastic collisions to the total energy balance was small (less than 10%). It means that even in such a light gas, such as helium, a pressure interval exists, in which such an approach, when the EDF is local, but the energy balance is determined by the inelastic collisions, is applicable.

At higher pressures the energy balance is determined by the (quasi)elastic collisions, and with the exception of the small vicinity of the excitation threshold, the EDF is given by

$$f_0^{(0)}(w,r)=Cn(r)\exp\left(-\int_0^w\frac{V_\varepsilon(w')}{D_\varepsilon(w')}dw'\right). \tag{43}$$

The radial anisotropy, as Eq. (41), is described by

$$f_{1r}(w,r)=-\frac{\nu}{\nu}C\exp\left(-\int_0^w\frac{V_\varepsilon(w')}{D_\varepsilon(w')}dw'\right)\times\left(\frac{dn}{dr}-\frac{V_\varepsilon}{D_\varepsilon}neE_r\right). \tag{44}$$

The ambipolar radial field corresponds to the absence of the total radial flux:

$$eE_r(r)=\frac{d\ln n(r)}{dr}\frac{\int_0^\infty\exp\left(-\int_0^w\frac{V_\varepsilon(w')}{D_\varepsilon(w')}dw'\right)\frac{\nu^3}{\nu}d\nu}{\int_0^\infty\exp\left(-\int_0^w\frac{V_\varepsilon(w')}{D_\varepsilon(w')}dw'\right)\frac{\nu^3}{\nu}\frac{V_\varepsilon}{D_\varepsilon}d\nu}. \tag{45}$$

Substituting the field (45) into Eq. (44), one has

$$f_{1r}(w,r)=-C\frac{\nu}{\nu}\frac{dn}{dr}\exp\left(\int_0^w\frac{V_\varepsilon(w')}{D_\varepsilon(w')}dw'\right)\left(1-\frac{\int_0^\infty\frac{w'dw'}{\nu(w')}\exp\left(-\int_0^{w'}\frac{V_\varepsilon(w'')}{D_\varepsilon(w'')}dw''\right)}{\int_0^\infty\frac{w'dw'}{\nu(w')}\frac{\delta(w')}{\delta(w)}\frac{\nu(w)}{\nu(w')}\exp\left(-\int_0^{w'}\frac{V_\varepsilon(w'')}{D_\varepsilon(w'')}dw''\right)}\right). \tag{46}$$

For atomic gases with constant $\delta(w)$ and for the power approximation $\lambda\propto v^\kappa$, the radial anisotropy vanishes at $\kappa=1$ when the $f_0^{(0)}(w)$ has a Maxwellian dependence. For $\kappa>1$ the differential flux is directed outward at large velocities [with respect to the average thermal one $\tilde{v}\sim\sqrt{D_\varepsilon/(mV_\varepsilon)}$] and directed inward at small v. If $\kappa<1$, the situation is inverse—the energetic electrons flow inward and the slow ones outward. Only in this case is the traditional fluid approach [9–11] valid.

VIII. ENERGY FLUXES

The results presented about the EDF anisotropy contain the complete information about the radial redistribution of particles and energy. It can be clearly seen that any attempts to describe such a complex phenomenon in terms of the traditional fluid approach, i.e., in terms of unidirected particles and energy fluxes, can lead to oversimplified and even to erroneous conclusions. This approximation corresponds to a description of the EDF anisotropy, which in fact depends in a complex way on the energy and radial position, in terms of its two first moments—the total particles and energy fluxes. Such an approach gives some idea about the real EDF anisotropy only in the case of unidirected differential fluxes. But if, as it is in our examples, in some spatial points the differential fluxes in several different energy intervals are oppositely directed, this approximation becomes inadequate.

Since the electron energy relaxation length $\lambda_\varepsilon=\varepsilon_1/(eE_z)$ of the EDF body is of the order of (or even exceeds [6,23]) the relaxation length of the EDF, the energy fluxes depend crucially on the details of the electron kinetics. Generally speaking, in the absence of frequent electron-electron collisions, which facilitate Maxwellian EDF's on short (with respect to the energy relaxation ones) spatial and temporal scales, the energy fluxes altogether cannot be obtained in the fluid approach. If the local energy input, for instance, takes place in a restricted region of energy space, between ε and $\varepsilon+\Delta\varepsilon$, the energy flux is transported on a scale of the order of λ_ε only by electrons in the same energy interval $\Delta\varepsilon$. The fluid approximation, which operates with the average (over the EDF) fluxes, energies, etc., results in such situations in crude mistakes.

In numerous publications (see, for example, [12]), the division of the energy flux into convective, diffusive, and conductivity terms, each of which is transported by the whole ensemble of electrons, was proposed. Such an approach is valid only for the local case described at the end of the preceding section, when the energy balance is dominated by

local quasielastic processes and the role of radial energy fluxes is small. Even in the local case, when the energy balance is determined by inelastic collisions with a large energy loss, the energy fluxes are not described simply by a $\nu(\omega)$ dependence. They cannot be found in the fluid approach [9–11].

In the simplest case of inert gases, when ionization and excitation energies are relatively close to each other, the resulting energy flux depends crucially on the relation between the frequencies of these processes. If the removal of excited and of metastable atoms is controlled by stepwise ionization, the inward-directed energy flux due to inelastic collisions is compensated (integrally over the whole tube cross section) by the energy outflux to the tube walls at $\varepsilon > e\Phi_w$. In the total balance the outward-directed energy flux of slow electrons dominates. Of course, this does *not* imply that this is true at every single point of the tube cross section.

On the other hand, if the excitation energy losses dominate over the ionization ones, the energy outflux to the tube walls is small. Since the net particle flux is absent, the inward-directed energy flux, which is transported by the energetic electrons, dominates over the outward-directed energy fluxes of both slow and fast escaping electrons. In some sense this corresponds to diffusive heating [13]. It seems more natural, especially when the EDF is nonlocal, to subdivide the total energy fluxes into the parts which correspond to different sections of the EDF, in which differential fluxes are oppositely directed. In Fig. 5 an example of such a division of the energy flux is given. It is clearly seen in Fig. 5 that the inward-directed energy flux of the fast electrons with $\tilde{\varepsilon} > 1$ is almost counterbalanced by the outward-directed flux of the slow ones. The resulting small energy flux is inward directed practically everywhere over the tube cross section. Only directly near the wall does a small outward-directed energy flux, which is connected to the electron escape to the wall, remain.

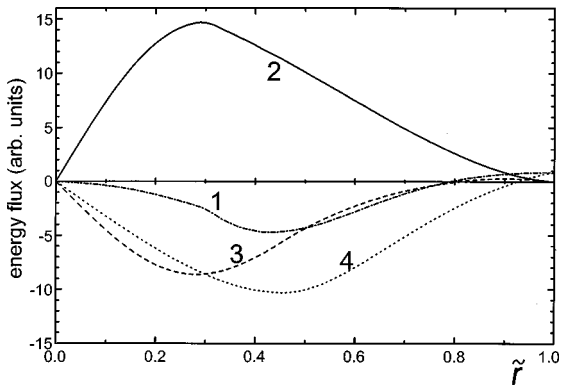

FIG. 5. Conditions of [7]: radius 1.7 cm and 100 Pa neon. (1) Total energy flux. (2) Energy flux of slow electrons with $0.1 < \tilde{\varepsilon} < 1$. (3) Energy flux of electrons with $1 < \tilde{\varepsilon} < \tilde{\Phi}_w$. (4) Energy flux of electrons with $\tilde{\varepsilon} > \tilde{\Phi}_w$. It is negative due to inelastic losses in the bulk plasma and changes its sign near the wall.

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